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PART I

THE IB PHYSICS IA GUIDE

1. GENERAL INTRODUCTION

The Internal Assessment (IA) is an important part of your Physics IB course. Not only does it count as 20 % of your final grade (!), but it is also an **opportunity** for you to explore some physics-related concepts that you found interesting during your classes.

The Physics IA gives you a chance to experience what being a scientist really entails: asking questions and designing experiments that try to answer them. This is at the root of the scientific method! What moderators look for in a Physics IA are signs of an inquisitive mind and evidence of the student's ability to conduct an accurate experiment while making pertinent conclusions.

An inquisitive mind means that you ask yourself about the **relationship** between two different variables and are able to design an experiment to **test** what that relationship is. For the experiment to be relevant, you need to make sure that your procedure is precise and accurate, and that you are able to handle errors well. Finally, you should be able to consider the **limitations** of your experiment when you discuss your conclusions.

Remember: There is nothing wrong if your experiment does not work out – the important thing is that you are **critical** about your findings, being able to explain why it did or did not work.

The Physics IA is very similar to what a "lab report" would be at university and consequently, the **formatting** and the rigour when **handling errors** are a very important part of it. In order to improve your presentation, you might want to use LaTeX but word will work too.

It does not matter which graphing software you use but you need to make sure you are comfortable adding errors and lines of best fit to your plots. Some suggestions for the software you might use are:

Presentation	**Software**
Typesetter	Word, Pages, LaTeX
Graphs	Excel, CurveExpert, OriginLab, LoggerPro
Plots	Desmos, Geogebra
Data Analysis	Audacity (sound), Logger Pro (motion)

There is no word limit in your IA but keep it between 6-12 pages (not including bibliography or appendices). Any additional data tables or graphs which are not essential to understand your work should be included in the appendices.

The IA is **internally assessed** so make sure you check with your teacher if there is any format and page limit that he/she would prefer you to follow.

Use your laboratory time as an **opportunity** to develop the skills you will need to use when carrying out your IA. Finally, keep in mind that completing a great IA would also help you with the data analysis questions in **Paper** 3!

2. HOW TO FIND AN INTERESTING TOPIC

Now you know what to do, you might be wondering "how do I pick the right topic?". Start thinking about the things you are **interested in** (e.g, music, sports, etc), and make a list of the topics you found the most interesting from the Physics syllabus. Next, do a bit of research about those topics and pick a law that **relates two variables in a simple way.** If you do not find a specific law, try to see if you can explain what the relationship between the variables should be and why.

The simpler the better: It is not about making a crucial finding but rather about being able to conduct an accurate and precise experiment while processing your data and interpreting your results correctly.

The most important thing is that the relationship you are studying is **crystal clear** and of some physical relevance. Therefore, you should put a lot of thought when choosing the **variables** that you will be studying. Let's recall few important things about physical variables: An experiment has a **dependent, independent** and **controlled** variables:

Variable	**Explanation**
Independent	The variable you change in a controlled fashion
Dependent	The variable that it is affected by the change in the independent variable
Controlled	Every other physical quantity that can alter the outcome of the experiment

When deciding on your IA topic, you should pick something **concrete, realistic** and of **interest** to you. Ideally, you want to go a bit beyond your syllabus to show **curiosity, initiative or independent thinking.** For example, if you learned about refractive index and Snell's law in class, you might want to study how the **refractive index (dependent)** of a material changes with its **temperature (independent)**.

Keep in mind: The dependent or independent variable might not be the things that you measure, but rather **what you calculate with your measurements.** In the previous example, the independent variable is the temperature (which you can easily measure with a thermometer) but the dependent variable is the refractive index, **which you can't measure directly.** In fact, the refractive index needs to be **calculated** using the incoming and outgoing angles that you **measure** with your experiment and Snell's law.

To give you an idea of what makes a good research question, here are some examples:

- *"How does the total weight of a bike relate to its stopping distance?"*

- *"How does the drop height of a ball affect the range (as measured from the center of the bucket) to the water splashing out?"*

- *"How does the hanging mass at the end of a cantilever affect the declination?"*

As you can see, they are all simple ideas that try to test the relationship between two quantifiable variables. Nevertheless, if you are feeling lost and do not know where to start, take a look at the resources in Section 5, which contain many ideas that you can use for inspiration.

Once it is clear which variables you are using, and what the relationship you expect them to follow is, you need to carry out an experiment to test whether the relationship between the variables holds or not. It is very important that your experiment shows a **functional relationship** between these two variables, **not a bar chart**. This means that your experiment should produce a function at the end, where in the x-axis you have your independent variable and in your y-axis you show your dependent variable. Keep this in mind when thinking about what variables to choose!

When carrying out your experiment, you need to design a systematic method that allows you to **measure and vary** your independent variable, and allows you to **accurately measure** how your dependent variable changes. **One of the most important things** is to make sure that your experimental method is designed to keep all the controlled variables **constant.** When I say **all the variables**, I mean it. Identifying all the relevant controlled variables is very important since they will be directly related to the source of systematic errors in your experiment and it will improve the accuracy of your experiment.

An example: If you want to measure the relationship between the bounce height of a basketball as a function of the height at which you drop it, one might think that the only controlled variable is the type and weight of the ball you use. But there are many more: The way the ball travels (e.g. vertically), the pressure inside the ball, the way in which you dropped the ball ,etc. Your experiment needs to make sure all these variables are consistent.

In conclusion, when choosing a topic, the most important thing is to ask a **clear** research question and to pick the **right variables** which you could **realistically measure** given the equipment you have.

3. BREAKDOWN OF THE GRADING MATRIX

The IA has a total of **24 marks** and five different criteria:

- Personal Engagement (2 marks)
- Exploration (6 marks)
- Analysis (6 marks)
- Evaluation (6 marks)
- Communication (4 marks)

Let us understand what is relevant to achieve full marks on each of the grading criteria.

3.1 Personal Engagement

In order to show personal engagement, you need to 1) explain **why the project is relevant to you** and 2) make sure that throughout your IA, you show that you are **creative** about what to study and how you carry out the experiment, demonstrating **critical thinking** by **questioning and justifying your experimental procedure.**

You can show personal significance, interest and curiosity in the introduction when you explain why you picked that particular research question for your investigation. For example:

"I am a clarinetist and so any scientific study of the sounds that instruments make is significant to me. I was interested in the physics behind musical instruments and decided to combine previous study of standing waves in closed pipes with an investigation into different wavelengths that can be played on the clarinet."

or

"This experiment will investigate how a liquid's surface area to volume ratio affects its rate of cooling. This is relevant in many real world situations, such as how much power swimming pool heaters use, or how long it will take to boil a pan of water – I was inspired to investigate this because I drink lots of tea. It is intuitive that a wider mug of tea will cool more quickly than one with a narrower neck, but I have always wondered just how much the surface area affects how quickly it cools. "

Independent thinking and initiative can be shown in different sections. It could be displayed, for example, in the way you design and carry out your experiment, or by being creative about the suggestions that could improve it. Finally, you can show independent thinking by coming up with the experimental method yourself, instead of following something that your teacher gave you or that you found online. If the different steps of your experiment are well justified, it shows that you have thought about it!

3.2 Exploration

The exploration is about how clear the research question is and how you carry out your experiment in order to get **precise** and **accurate** results. This is an important part of the grade and it is divided in three different aspects:

1.1. Research question.

The title has to be precise (it is clear what the dependent and independent variables you are studying are). This also has to be written clearly in the introduction. For example:

"How does the time that the free end of a cantilever takes to oscillate 15 times depends upon its free length?"

Moreover, you must give the relevant **background**. In the example above, one should talk about moments of inertia, how they relate to the oscillation, and the length of the cantilever. Remember, you don't have to copy your whole Physics textbook, you just need to give **enough information** in order to understand the **context** and **relevance** of the investigation. After writing the background, you should include the **hypothesis** that you expect your variables follow, which should easily be understood.

Note: You might need to conduct research to make sure your hypothesis is meaningful.

1.2. Variables, variables and more variables.

You need to write down your dependent, your independent variables and **a list** of your controlled variables. For the dependent and independent variables, you also need to include the **units**[1] **in which you measure them** and the **range** you will use:

Independent: *Free length of the cantilever (cm).*
Dependent: *Time taken by the free end of the cantilever to oscillate 10 times (seconds).*
Controlled:

- *Kind of cantilever used.*
- *Orientation of the cantilever when it is fixed (Perpendicular in relation to the table).*
- *Number of oscillations for which the time is recorded (10).*
- *Amplitude of oscillation of the cantilever (3cm).*
- *Type of motion sensor used. (Vernier Lab Pro).*
- *Recording sample interval for the motion sensor and number of samples. (0.1 seconds, 300 samples).*

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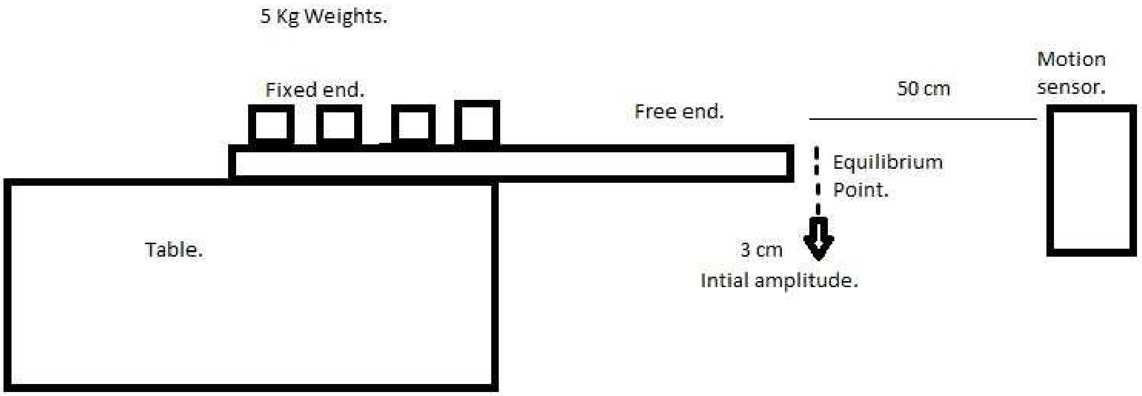
[1] If one of your variables has no units, you can label them as Arbitrary Units (A.U.).

- *Point and distance at which the motion sensor is kept in relation to the free end of the cantilever. (Equilibrium point and minimum distance needed for the cantilever to record data).*
- *Way of keeping the end of the cantilever fixed and relative position of different fixed ends. (Same straight line).*
- *The place where the experiment is conducted.*
- *Ambient conditions like wind, atmospheric, pressure, humidity and warmness of the air.*

2.1. Experimental set-up.

You should give a list of **all** the apparatus that you will use to carry out your experiment, including their experimental **limit of reading** when relevant[2]. Also, you should include a **sketch** to show the experimental set-up. It does not have to be something sophisticated, just a simple diagram or a picture where you label the different apparatus. This helps to visualize how the experiment is carried out. If you take the diagram from somewhere, **make sure you cite your source,** but it is better **if you make it yourself**.



2.2 Experimental method.

In your experimental method section, it is important that you write a **clear list of steps** – almost like a cooking recipe, so that any other student in a similar high-school lab could take your method, repeat the same experiment and get the exact same results as you. **The most important thing** is that your method ensures that you keep your control variables constant:

"<u>Step 5.</u> Connect the motion sensor to the computer and set the data collection for a number of 300 samples with a recording sample interval of 0.1 seconds.

<u>Step 6.</u> Use the ruler to measure a distance of 3cm downwards from the free end point of the cantilever in a perpendicular direction in relation to the position of the table, move the final part of the free end of the cantilever that distance with your finger as it is shown in Figure 2 and hold it at that point, being careful to not displace the fixed point from where it is. In this way, the initial amplitude of oscillation of the cantilever is controlled."

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[2] The limit of reading is the smallest value your apparatus can measure. For a standard ruler this is usually 1 mm. This is important because one usually approximates the uncertainty in the quantity you measure with a particular apparatus as ½ the limit of reading of the apparatus.

See how the number of samples, the recording time interval and the way of setting up the initial amplitude of oscillation **are indicated and kept under control.**

In your method, it also needs to be clear how you are **measuring** and **recording** your dependent and independent variables, and which instruments you are using to measure them:

"__Step 7.__ Start the data collection at the same time you stop holding the free end point of the cantilever.

__Step 8.__ On the computer you will see a graph of displacement versus time. Count 10 oscillations and record the time needed to complete those oscillations in the table below for Trial 1 in sample 1."

Finally, you might need to include a statement about safety, ethical or environmental issues, but **only if they are relevant** to your investigation. If not, just write a sentence saying that there were not significant issues.

3.1 Make sure you make **enough variations** of your independent variable (**5 at least**) within the appropriate range and explain how you are **changing** it in your method. This will allow you to accurately observe the trend that you are trying to test. In order to figure out the appropriate range, you could perform a simple, preliminary experiment to find out what the relevant ranges for the dependent and independent variables should be. If this is not necessary, make sure you include a **justification** of why you are using a particular range.

3.2 **Repetitions.** You need to repeat your experiment **three times** so that you have three different measurements of your dependent variable for each value of your independent variable. **These instructions must be included in the method as well:**

"__Step 11.__ Repeat steps 3-7 three more times keeping the same fixed point in the line drawn in the table using the same cantilever and record the time taken to oscillate 10 times in the table below for Trial 2 and Trial 3.

__Step 12.__ Repeat steps 3-11 setting the fixed end at the different points 2 to 7. Repeat the whole process one time for each new fixed end using the same cantilever and record the time taken by the cantilever to oscillate 15 times in the table below for samples 2 to 7."

Remember: What you measure is your **raw data** and should therefore be included in a **raw data table.** What you plot during your analysis is your **processed data,** and you need to include a different **processed data table** for it.

3.3 Analysis

The analysis section is about converting your raw data into some **quantitative statement** about your research question. Handling of **errors** and **sample calculations** are key here! As before, this criterion is divided into three sections.

1.1. **Raw data.** Your raw data table must have: A clear **title**, the **measured** variables for all three trials (including the **label** of the variables), the **units** and the **uncertainty** in your measurements[3] to **1 Significant Figure.** Make sure that your recorded data has the same number **of decimal places as your error for each variable!**

For example, in an experiment where laser light is shined through a small slit to study the relationship between the size of the diffraction pattern's maxima and the distance between the slit and the screen where the pattern is recorded, a raw data table would look like:

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[3] Remember this often depends on the limit of reading of the apparatus that you are using.

Table 1. Raw Data table including the measured size of the maxima (2y) in cm with three repetitions for each sample at different distances between the slit and the screen (D) in cm.

Sample	*Slit width (b) / · 10^2 cm ± 0.1 · 10^2 cm*[4]	*Distance (D) / cm ± 0.1 cm*[5]	*Size of the maxima (2y) / cm ± 0.05 cm*[6]		
			Trial 1	*Trial 2*	*Trial 3*
1	*4.7*	*150.0*	*1.25*	*1.30*	*1.20*
2		*160.0*	*1.25*	*1.35*	*1.30*
3		*170.0*	*1.35*	*1.30*	*1.50 **
4		*180.0*	*1.40*	*1.40*	*1.35*
5		*190.0*	*1.40*	*1.45*	*1.45*
6		*200.0*	*1.50*	*1.50*	*1.75 **

Note. Values labeled with a star in sample 3 and 6 seem to have a large random error as they differ from the other three values in sample 3 and 6 by more than 10% of the measurement. These values will therefore be ignored in subsequent processing.

Notice how in the example above the errors are given to 1 significant figure and the decimal places for the recorded values match the decimal places of their corresponding uncertainties. **This is very important.** In addition, if you have not already explained the uncertainties in your dependent or independent variables, this is the time to do it (see footnotes in Table 1). Finally, include a note about **any particular data points which behaved strangely** (called outliers) so that later, in your evaluation, you can refer to them and explain them. This shows **critical thinking!**

1.2. **Average of trials.**

You should add an additional table for the average in your dependent variable. Make sure you include the uncertainty in the average **with one significant figure** and that the number of decimal places in this uncertainty is consistent with the number of decimal places of the averages.

See in the table below how the uncertainty in the average has one significant figure and two decimal places, so that the averages are given to two decimal places.

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4. Uncertainty in the slit width is taken to be the limit of reading of the screw gauge used, we are measuring the distance from the value to zero, both points with an uncertainty of LoR / 2, therefore, the total uncertainty is LoR /2 + LoR / 2 = LoR.
5. Uncertainty in the distance between the screen and the slit is taken to be the limit of reading of the rule used, since we are measuring the distance from the value to zero, both points with an uncertainty of LoR / 2, therefore, the total uncertainty is LoR /2 + LoR / 2 = LoR.
6. Uncertainty in the size of the maxima is taken to be the limit of reading of the rule used, since we are measuring the distance from the value to zero, both points with an uncertainty of LoR / 2, therefore, the total uncertainty is LoR /2 + LoR / 2 = LoR.
7. The screw gauge has been checked for zero error, giving a value of +1 · 1 / 100 mm, so the actual reading was 48 but the length of the slit is 47 · 1 / 100 mm.

4. THE RELATIONSHIP BETWEEN TEMPERATURE AND COEFFICIENT OF RESTITUTION

Relationship between temperature and coefficient of restitution

Research Question:

How does the temperature of a ball affect its coefficient of restitution when dropped from a fixed height of 1m?

Variables:

Input: Temperature of Ball
Output: Coefficient of Restitution

Background information:

During this experiment, the output or dependent variable will be the coefficient of restitution. The coefficient of restitution is a method through which the elasticity or restitution of a collision can be mathematically demonstrated. For the collisions of two moving objects, it is defined as the ratio of the difference in the bounce velocities and the difference in the approach velocities.[1]

$$\boldsymbol{C_r = \frac{u_a - u_b}{v_a - v_b}}$$

$\boldsymbol{u_a = bounce\ velocity\ of\ object\ a}$
$\boldsymbol{u_b = bounce\ velcoty\ of\ object\ b}$
$\boldsymbol{v_a = approach\ velocity\ of\ object\ a}$
$\boldsymbol{v_b = approach\ velocity\ of\ object\ b}$

If the collision is perfectly elastic, the coefficient of restitution will be 1. For real-world, inelastic collisions, the coefficient will be between 0 and 1. As this experiment deals with the collisions of a moving object, the ball, with a stationary object, the ground, a simplified version of this equation can be used:

$\boldsymbol{C_r = \frac{u}{v}}$	$\boldsymbol{u = ball's\ bounce\ velocity}$ $\boldsymbol{v = ball's\ approach\ velocity}$
$\boldsymbol{C_r = \frac{\sqrt{2gh}}{\sqrt{2gH}}}$	If the ball is dropped from rest and, when it comes into contact with the ground, comes to rest for a fraction of a second, the equation of motion $v = \sqrt{u^2 + 2gh}$ can be applied but as the initial velocity is 0, it is quite simply: $v = \sqrt{2gh}$ $h = ball's\ bounce\ height$ $H = ball's\ drop\ height$
$\boldsymbol{C_r = \frac{\sqrt{h}}{\sqrt{H}}}$	After simplifying this the final equation includes only the height of drop and height of bounce.

The input variable or dependent variable will be the temperature of the ball. As we are dealing with a ball filled with a gas, air, we can discuss it in terms of the ideal gas law:

$$Pressure \times Volume = Number\ of\ moles \times Gas\ Constant \times Temperature$$

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[1] Adli Haron and K. A. Ismail. "Coefficient of Restitution of Sports Balls: A Normal Drop Test." *IOP Conference Series: Materials Science and Engineering* 36.1 (2012): n. pag. Web. 7 May 2016

In this experiment, we assume that the volume will stay constant while the temperature changes. For this to happen the pressure must change too and in this case the pressure is proportional to the temperature. The change taking place between trials (as the ball's temperature changes) is isochoric – work being done on the system causes the pressure and temperature to change but the volume remains constant. The kinetic energy being transferred to the ball through the temperature it changes it is exposed to is equal to the work done.

It is clear that the lower the pressure inside the ball the easier it is to deform the ball during the bounce as pressure is the force exerted per unit area. It is this outwards force which allows the ball to hold its shape and thus resist deformation. If the ball is easier to deform, the bounce will take a longer amount of time. When considering the momentum of the ball during collision, and noting that: $F = \frac{\Delta p}{t}$ and if the change in momentum is equal for every bounce, as the drop velocity and mass will be the same, the force is inversely proportional to the time of contact. Therefore, a greater force will be exerted by the ball onto the ground when the time of contact is shorter. Applying Newton's Third law, the amount of force exerted by the ball onto the ground is equal to that of the force exerted by the floor on the ball.

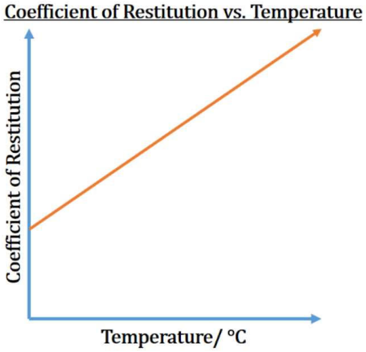

Figure 1: Predicted Relationship

Therefore, the force exerted by the floor is inversely proportional to the time of contact and as the contact time is inversely proportional to the pressure of the ball, the pressure of the ball is proportional to the force exerted by the ground upon impact. Overall, the force supplied by the ground is proportional to the bounce height and so it can be inferred that the higher the temperature, the higher the coefficient of restitution. The predicted relationship is noted in figure 1, it does not go through the origin as this would represent a coefficient of restitution of 0, which is impossible as that would mean no energy remains after the collision whatsoever.

Control Variables:

	Why it has to be controlled?	How it is going to be controlled?
Ball's initial air pressure	As the temperature is changed, the pressure of the ball will change but the pressure of the ball at room temperature will have to be constant for every trial. If initial pressure is higher for a particular trial, the results will be skewed towards a higher coefficient of restitution.	Before placing the ball in the bath of water to change its temperature, the pressure will be noted and adjusted.
Ball's volume	In the ideal gas law, for the temperature to be proportional to the pressure, the volume must not change. Between trials, if the volume has changed, the temperature will have a disproportionate effect on the pressure and that will affect the rebound height.	The same ball will be used during each trial with a fixed diameter.
Bounce Surface Elasticity	The transfer of energy during the collisions depends on the elasticity of surface that the ball is bouncing upon. Using difference surfaces will cause different	Each trial will be undertaken on the same tiled flooring.

	elasticities and therefore will skew the bounce height and the coefficient of restitution	
Elasticity of the ball's material	Similarly, as with the bounce surface, the energy transfer depends on the material with which the ball is made. If the material is more elastic, it will allow for compression and retain more energy. As the balls, will be placed in water, there is the problem of them being wet which will affect the bounce height and yield lower values for the coefficient of restitution.	The same ball, will be used for each trial and it will be dried when taken out of the water with a dry towel.
Bounce height	The coefficient of restitution depends on the drop velocity of the ball. As this is defined by the square root of twice the height drop times the gravitational field strength, therefore ensuring the height dropped is the same will ensure the velocity is the same too.	The ball will be dropped from 1 m each time.

Method:

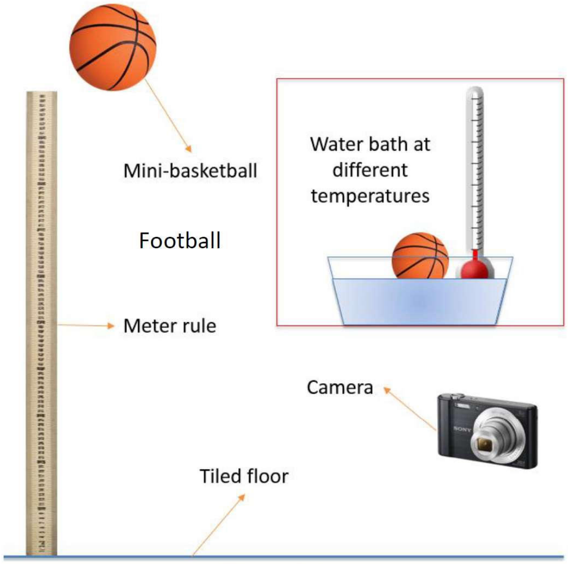

Figure 2: Setup

1. Pump the football with an air pump until a pressure of 135kpa is achieved, use a pressure sensor to check for this value.
2. Record the room temperature.
3. Place meter rule vertically using clamps stand to hold it up.
4. Place a camera 1.5m from the ruler and 0.5 m above the ground as to reduce parallax error.
5. Hold ball so that the bottom of the ball is at the 1 m mark.
6. Start recording with the camera.
7. Drop the ball and allow it to bounce at least once.
8. Place ball in baths of water at 30°C, 40°C, 50°C, 60°C, 70°C, 80°C and repeat steps 1-6.
9. Record the pressure of the ball after it has been removed from the water with a pressure sensor.
10. Record the temperature of the balls after it has been removed from the water with an infrared temperature sensor.
11. Quickly wipe the ball with a dry towel and then drop it from 1 m.
12. Repeat steps 8-10 3 times for each temperature (to obtaining three trials).
13. Place the ball in a box of dry ice and record the temperature of the dry ice, repeat steps 1-6 3 times for this temperature (to obtain three trials).
14. Record the pressure of the ball with a pressure sensor after it has been removed from the dry ice.
15. Record the temperature of the ball with an infrared temperature sensor after it has been removed from the dry ice.
16. Drop it from 1 meter and then repeat steps 12 and 13 3 times (to obtain three trials).

Safety:

Risks of burns: using water at 60°C and above can cause burns to skin. Using gloves or tongs when placing or removing the ball in the water bath is advisable at these high temperatures to reduce this risk.

Dry Ice: this substance is extremely cold, around -79°C, there always use caution when handling it. Wear gloves and never expose skin directly for a pronlonged perood as this can freeze cells an burn the skin. Also make sure to use the dry ice in a well ventilated room or in a fume cupboard as it gives off a lot of carbond dixide which can be dangerous.

Data collection:

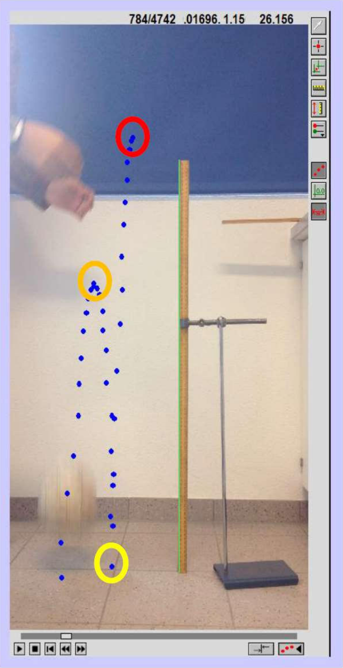

Figure 3: Logger Pro Video Analysis

Using Logger Pro, and the videos obtained during the experiment, the values needed to calculate the coefficient of restitution, namely the height at which the ball is dropped and the height the ball bounces, can be extracted. As a meter rule was placed in the shot, this can be used as a scale on Logger Pro as to be able to obtain distance value, an example can be seen in figure 2.

This produces a graph showing the full motion of the ball, both horizontally and vertically (shown in figure 4), however we are only interested in specific values: the drop height (circled in red), the height of the floor (circled in yellow) and the height of the bounce (circled in orange).

This method of collecting data which will be used to calculate the coefficient of restitution, has some limitations. Firstly, the camera used is an iPhone 5 and has a capability of 30fps meaning that if the ball were to have reached its peak and fallen within 1/30 of a second, it would not be recorded by the camera and therefore the peak recorded would be lower than the actual peak. This issue is also faced when determining the floor value as it might reach the floor and bounce back, all within 1/30 of a second and thus would not yield accurate results.

Another limitation stems from the use of a ruler as a scale. The ruler must be manually highlighted on Logger Pro for a scale to be created but this means that it depends on how exact this manual selection is done. Additionally, the position of the ball is manually denoted, adding a similar problem as above. An uncertainty of ±0.005m is assumed for the ruler itself as this is half of the smallest division but assuming this for the distances generated by Logger Pro is an over simplification as the uncertainty here stems from not only how precisely the scale was selected but also how accurately the positions of the balls were recorded. This being said, there is no comprehensive way of obtaining an accurate uncertainty due to those parameters and thus the uncertainty of ±0.005m will be used in the data analysis.

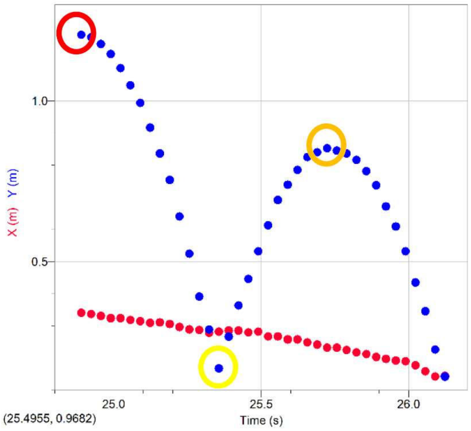

Figure 4: Motion Graph of Ball

5. HOW DOES THE MASS OF THE COUNTERWEIGHT USED TO LAUNCH THE PROJECTILE OF FIXED MASS AFFECT THE RANGE OF THE PROJECTILE?

INTRODUCTION

Catapults are ancient Roman weapons that launch projectiles at their enemies using a large weight to project it. Earlier in the year, I had to build a catapult during a leadership camp as part of a team building activity. Though we added an object of very large counterweight to project a projectile of very small mass, the object did not fly as far as expected. However, one of the other groups used a less heavy counterweight and their projectile flew further. This made me want to investigate the effect of the mass of the weight to the range of the projectile.

RESEARCH QUESTION

To investigate the relationship, I came up with my research question. "**How does the mass of the counterweight used to launch the projectile of fixed mass affect the range of the projectile?"** I will be investigating the effect on a projectile of fixed mass so as to be able to establish a relationship based on just two variables, so that a future trend can be predicted.

Independent Variable: The mass of the counterweight used to launch the projectile, defined as M_c. An electronic mass balance will be used to measure the pieces of plasticine used as the counterweight. The range of masses will be from 400 g to 850 g in 50 g intervals.

Dependent Variable: The range of the projectile, measured by recording the initial point of landing on the floor after the projectile is launched to the point from which it was launched. *(See diagram 1 for the labeled diagram of catapult.)*

Controlled Variables: *(See diagram 1 for the labeled diagram of catapult.)*

1. Mass of the projectile, controlled by using the same projectile of (100±1) g throughout the experiment. Controlled so that the relationship is strictly between the mass of the counterweight and the range.
2. Height of catapult, controlled by using the same catapult. Controlled because the initial height of launch will affect the range.
3. Height of pivot, controlled by using the same catapult. Controlled because the initial height of launch will affect the range.

4. Initial angle of launch, calculated to be 57.1°, controlled by using the same catapult. Controlled because the initial angle of launch will affect the range.
5. Initial distance between the projectile and the pivot, controlled by using the same projectile. Controlled because a change in the distance will affect the moment about the pivot, which will in turn affect the force with which the projectile is launched and hence, the range.
6. Distance between the counterweight holder and the pivot, controlled by using the same catapult. Controlled because a change in the distance will affect the moment about the pivot, which will in turn affect the force with which the projectile is launched and hence, the range.
7. Air resistance, controlled by conducting the experiment in the same place with the fans off and the windows closed. Controlled because wind on the projectile will affect the range.

THEORY

Levers

A catapult falls under one of three classes of levers, specifically to the first class of levers, the type where the fulcrum or pivot is placed in between the load and the effort, in this case the projectile and the counterweight.[1] Due to the position of the load and effort, they will move in opposite directions about the pivot. This will cause a change in the net moment about the pivot when the mass of either objects or the distance of the objects from the pivot changes. The moment about the pivot can be calculated through the equation (Moment) = (Force) x (Distance from Pivot). In the case of this catapult, the projectile and counterweights are equidistant from the pivot and hence, the moment about the pivot is determined solely by the force at either end, (since the mass of the ruler and the support beam is constant, and the pivot is in the middle of the ruler for this setup).

Forces and Energy Conversion

The force can be calculated to be a product of the mass and the acceleration, which in this case is the acceleration due to gravity, a constant that is equal to 9.81 m s^{-2}. Since the

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[1] "Design and Technology. Mechanisms." *BBC Bitesized*. BBC, n.d. Web. 5 May 2016. <http://www.bbc.co.uk/schools/gcsebitesize/design/systemscontrol/mechanismsrev1.shtml>.

acceleration is constant for both the masses, and since it will not affect the horizontal velocity and hence will not affect the range of the projectile, the force is dependent on the mass of the object. It can be seen that the moment about the pivot is dependent on the mass of the counter weight since the mass of the projectile is also constant. Therefore, in this specific case based on the class of the lever and the design of the catapult, the mass of the counterweight must always be greater than the mass of the projectile launched. It is generally assumed that with an increase in the mass of the counterweight for the same projectile, the range of the projectile will also increase. With a larger mass of the counterweight, there will be a larger net momentum, resulting in a higher initial velocity of the projectile. But is there a maximum limit to the increase in range? After a certain point, the range may begin to decrease again. Can the maximum range of the projectile be found to optimize the mass of the counterweight?

Newton's Laws of Motion

According to Newton's First Law, the horizontal velocity of the projectile will be constant since there is no horizontal external force exerted on it, assuming that there is negligible air resistance, and hence, the range will not be affected. According to Newton's second law, there will be a constant force exerted on the vertical component of the velocity of the projectile due to the constant acceleration. With a combination of these two components, we see that the projectile will be launched and follow a parabolic path on the air. Using kinematics equations, the range of the projectile can be calculated based on the initial velocity of the projectile.

The equation used is:

$$y = y_0 + (\tan(90-\theta))x - \left(\frac{\frac{g}{2}}{v_0^2\,(\cos(90-\theta))^2}\right)x^2 \qquad \text{(Equation 1)}$$

Where,

y = The height of the projectile at any moment, measured in meters, m.

y_0 = The initial height of the projectile at the moment of launch, measured in meters, m.

θ = The angle between the arm of the catapult and the floor, calculated by using trigonometric rations of the height of the pivot off the ground and it's distance from the initial position of the projectile before launch, represented in degrees, °.

x = The range of the projectile, measured in meters, m.

g = The acceleration due to gravity gravitational, 9.81 m s^{-2}.

v_0 = The initial velocity of the projectile at the moment of launch, measured in meters per second, m s^{-1}.

The equation above resembles a quadratic equation, so as to represent the parabolic nature of the motion of the projectile.

However, the motion of the projectile can also be explained through equations relating to energy conservation in the closed system. The energy gained of the projectile launched will be equal to the net energy lost from the counterweight when it is released, assuming that there is no friction in the system.

The gravitational potential energy lost from the counterweight can be calculated through the equation:

$E_1 = M_c \times g \times \Delta h$

Where,

M_c = The mass of the counterweight, measured in kilograms, kg.

g = The acceleration due to gravity gravitational, 9.81 m s^{-2}.

Δh = The height of the counterweight launcher from the ground, measured in meters, m.

The energy gained by the projectile can be calculated through the following equation:

$E_2 = (M_p \times g \times \Delta h) + ((1/2) \times M_p \times v_0^2) + ((1/2) \times M_c \times v_0^2)$

Where,

M_p = The mass of the projectile, measured in kilograms, kg.

g = The acceleration due to gravity gravitational, 9.81 m s^{-2}.

Δh = The height of the projectile launcher from the ground, measured in meters, m.

M_c = The mass of the counterweight measured in kilograms, kg.

v_0 = The initial velocity of the projectile at the moment of launch, measured in meters per second, m s^{-1}. The velocity of the projectile as it gains KE will be the same as the velocity of the counterweight as it loses GPE because of the fact that they are both equidistant from the pivot.

It can be assumed that the energy of the system is conserved, $E_1 = E_2$, ignoring rotational kinetic energy and assuming that the masses moved vertically.